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RESEARCH ARTICLE

Apply Bayesian Inference with Normal–Normal Conjugate to Forecast Renewable Energy Generation: A Case Study of Waste-to-Energy in Taiwan

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ABSTRACT

This paper applies Bayesian inference with normal–normal conjugate to forecast renewable energy generation. The generation forecasts a probability distribution rather than a quantitative value. An assumed normal distribution is initialized for renewable energy generation. This assumed normal distribution's parameters, the mean μ, and the standard deviation σ, are inferred by Bayesian inference afterward. However, applying Bayesian inference barely shall encounter an intractable integral. To circumvent the intractable integral, this paper considers the normal-normal conjugate method. This method fixes the assumed normal distribution's σ and characterizes μ as another normal distribution and then infers the latter normal distribution parameters. A case study of waste-to-energy generation forecast in Taiwan is investigated in this paper. It has been found from the investigation that the Bayesian inferred probability distribution outperforms the assumed one.

Index Terms—Bayesian inference, forecast, normal–normal conjugate, renewable energy generation, waste-to-energy

I. INTRODUCTION

Currently the global temperature keeps rising and causes many threats like warm house effect, extreme climate, and ecological disasters. The temperature rise is a consequence of vast carbon emissions from various vehicles, industrial factories, and electric power generation plants. They all consume a great amount of fossil fuel. In order to reduce fossil fuel consumption, engineers have developed renewable energy, including wind power, solar photovoltaic, biomass energy, and so on. Developing renewable energy not only reduces fossil fuel consumption but also creates an eco-friendly environment. Tremendous efforts have been contributed to developing renewable energy generation [1, 2].

Developing renewable energy is a remedy, but it also brings new challenges. Take wind power as an example. Sometimes the weather is windy so wind power is adequate. Sometimes the weather is calm so wind power shall be inadequate. Obviously, the renewable energy is intermittent. One of the challenges is transforming renewable energy into electric power and saving the electricity. Many electric energy storage schemes like superconducting material, super capacitor, batteries, and large-scale pumped storage scheme are underdeveloped [3-5]. Another challenge is forecasting renewable energy generation. Engineers attempt to forecast intermittent renewable energy generation accurately for the sake of operating the power system economically and reliably. Many methods of renewable energy forecasts or predictions have been studied. These methods include moving average (MA), auto-regressive MA (ARMA), artificial neural network (ANN), support vector machine (SVM), and hybrid intelligence [6–7].

Most of these methods forecast a quantitative value associated with a certain error. This paper provides another viewpoint. It forecasts a probability distribution of the renewable energy generation. The probability distribution is inferred by the Bayesian inference. It can be considered a data-driven approach. In literature, the Bayes theorem has been applied to many power system aspects, such as transient stability assessment [8], parameter identification of power system dynamic model [9], power flow analysis [10], power plant dispatch [11], optimal power load flow [12], and power system state estimation [13].

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This paper applies Bayesian inference with normal–normal conjugate to forecast renewable energy generation. To begin, an assumed normal distribution for renewable energy generation is initialized. This assumed normal distribution is characterized by two parameters, the mean μ and the standard deviation σ. This assumed normal distribution will be inferred by Bayesian inference afterward. However, using Bayesian inference barely shall encounter an intractable integral problem. To circumvent this intractable problem, Bayesian inference is aided by the normal–normal conjugate method in this paper. The normal–normal conjugate method first fixes the former assumed normal distribution's σ and characterize μ as another normal distribution. The latter normal distribution's parameters are called hyperparameters. Next, the normal–normal conjugate method updates hyperparameters analytically. The updated hyperparameters shall characterize the former assumed normal distribution parameter μ. Finally, the characterized parameter μ, together with the fixed parameter σ, constitutes the inferred normal distribution. This paper applies this inferred normal distribution to forecast renewable energy generation.

To testify the proposal, a case study of Taiwanese waste-to-energy power generation is investigated. Waste-to-energy is one of the renewable energy resources. In the renewable energy realm, much attention has been focused on wind power and solar photovoltaic generation. The waste-to-energy generation appears to be ignored in literature. This paper attempts to make up this ignorance.

II. BAYESIAN INFERENCE

A. Probability Distribution and Its Parameters

Probability distribution describes the possible values of a random variable. It also describes the probabilities of occurrence of those values. Parameters are the controllable parts of a probability distribution. Parameters decide the shape, or content, of a probability distribution. For a discrete random variable, probability distribution is discrete. It contains all values of probabilities that add up to one. For a continuous random variable, probability distribution is continuous. The area under a continuous probability distribution is one [14].

A classic example of discrete random variable is tossing a coin, which appears either head or tail. This example can be expressed by a binomial distribution, which is given in (1),

$$\Pr(y) = f(y;n,p) = C_y^n p^y (1-p)^{n-y} \quad (1)$$

In (1), $\Pr(\cdot)$ "means" represents the probability, and f means a function, where y is the number of appearing heads or tail. Binomial distribution is characterized by 2 parameters, n and ρ. n is the number of trial, and P is the probability of a head appearing. For example, the probability of obtaining two heads when tossing a fair coin (with $P = 0.5$) for three times is

$$\Pr(2) = f(2;3,0.5) = C_2^3 0.5^2 (1-0.5)^{3-2} = 3 \times 0.5^2 \times (1-0.5)^{3-2} = 0.375 \quad (2)$$

Binomial distribution can be presented simply by (3).

$$Y \sim \text{Binomial}(n,p) \quad (3)$$

Note that in (3), Y denotes the discrete random variable. That is, tossing three times a fair coin is represented simply by

$$Y \sim \text{Binomial}(3,0.5) \quad (4)$$

The random variable Y has values $y = 0,1,2,3$. And Pr ($Y = y$) is 0.125, 0.375, 0.375, and 0.125, respectively. Fig. 1 shows the binominal distribution of the results.

A special case of binominal distribution is the Bernoulli distribution. Bernoulli distribution indicates that n = 1. It means that if we toss a coin just one time and the coin is fair, then the probability of appearing head is

$$f(1;1,0.5) = C_y^n p^y (1-p)^{n-y} = 0.5 \quad (5)$$

However, if the coin is not fair, say, the probability of appearing head is 0.4, then we would have

$$f(1;1,0.4) = C_y^n p^y (1-p)^{n-y} = 0.4 \quad (6)$$

For a continuous random variable case, a well-known probability distribution is the normal distribution. It is also called the Gaussian distribution. It has two parameters, the mean μ and the standard deviation σ. The normal distribution is expressed by

$$P(x) = f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)} \quad (7)$$

It should be emphasized that the outcome is a probability density $P(\cdot)$, rather than a probability $\Pr(\cdot)$. Like binomial distribution, the normal distribution can be represented simply by

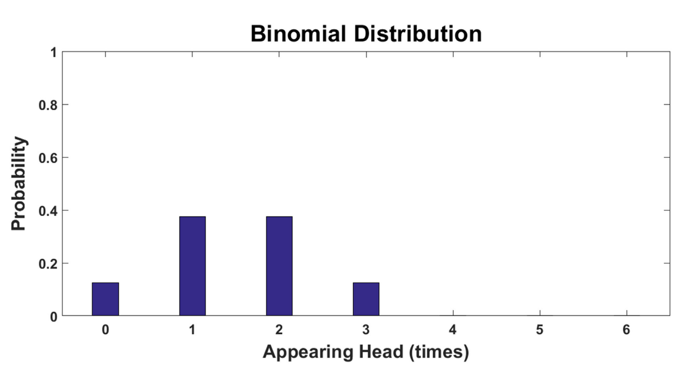

Fig. 1. Binominal distribution.

Main Points

- Apply Bayesian inference to forecast renewable energy.
- The Bayesian inference is aided with normal–normal conjugate.
- Forecast the waste-to-energy generation in Taiwan.
- Eighty percent accuracy of unforeseen data samples is tested.

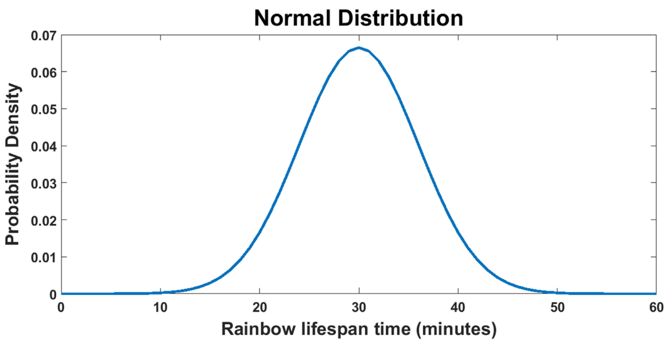

Fig. 2. Normal distribution.

$$x \sim N(\mu, \sigma) \tag{8}$$

Take the lifespan of a rainbow as an example. If a rainbow lifespan on the average is 30 minutes, and if the probability distribution's standard deviation is 6 minutes, then the normal distribution is shown in Fig. 2.

By means of the mean μ and the standard deviation σ, a normal distribution has the property of

$$\begin{aligned} &\mu \pm 1\sigma \text{ will cover a probability of } 0.6827. \\ &\mu \pm 2\sigma \text{ will cover a probability of } 0.9545. \\ &\mu \pm 3\sigma \text{ will cover a probability of } 0.9973. \end{aligned} \tag{9}$$

According to (9), it is commonly said that $\mu \pm 3\sigma$ will cover almost all the population [14].

No matter what type of probability distributions are, their parameters describe how the distributions look like. Different parameters lead to different distribution shapes. By tuning a distribution's parameters, the shape of the probability distribution will be changed.

B. Bayesian Inference

Before introducing the Bayesian inference, it is essential to revisit conditional probability. (10) tells the conditional probability that the probability of hypothesis A occurred, given hypothesis B occurred.

$$\Pr(A|B \tag{10}$$

Again, the term $\Pr(\cdot)$ indicates that it is a probability. The vertical bar means that the probability of hypothesis A given hypothesis B.

Extending conditional probability in (10), we yield the Bayes theorem [15],

$$\Pr(A|B) = \frac{\Pr(B|A)\cdot\Pr(A)}{\Pr(B|A)\cdot\Pr(A) + \Pr(B|\sim A)\cdot\Pr(\sim A)} \tag{11}$$

In (11), $\Pr(A|B)$ is the posterior probability of A given B, Pr (B|A), is the probability of B given A, and Pr (A) is the prior probability of A. It $\Pr(B|\sim A)$ is the probability of the B given ˜A. The symbol "~" means the logical operator "NOT." Pr(˜A) is the probability of ˜A. Assuming there are n hypotheses, and also assuming the i^th hypothesis denoted by H_i, we can extend the Bayesian theorem from (11) into

$$\Pr(H_i|\text{data}) = \frac{\Pr(\text{data}|H_i)\cdot\Pr(H_i)}{\sum_{i=1}^{n}\Pr(\text{data}|H_i)\cdot\Pr(H_i)} \tag{12}$$

(12) is used for discrete distribution case. In practice, a hypothesis H_i refers to a parameter value of the probability distribution. For the continuous random variable case, the probability distribution parameter is denoted by θ. The variable θ can be a vector if there are multiple parameters for the probability distribution. For the continuous probability distribution, we shall have infinite hypotheses for a parameter. Therefore, the Bayesian inference becomes

$$P(\theta|\text{data}) = \frac{P(\text{data}|\theta)\cdot\Pr(\theta)}{\int P(\text{data}|\theta)\cdot\Pr(\theta)d\theta} \tag{13}$$

It is worth pointing out that the term Pr (data|H_i) in (12) and the term P (data|θ) in (13) are called likelihood. The meaning of likelihood is very similar to that of probability, but likelihood and probability are different to a certain extent. Likelihood can be explained by that, given a probability distribution's parameter (or hypothesis) and also given the outcome of a random experiment, the likelihood is the probability of the experimental outcome under the hypothesis (namely, under the parameter). Table I summarizes the steps of applying Bayesian inference to a research topic.

Note that prior and posterior probability distribution functions are identical but their parameters are not. The parameters of prior and posterior probability are called hyperparameters. Likewise, the assumed and the inferred probability distribution functions are identical but their parameters are not. Hyperparameters characterize the assumed and inferred probability distributions' parameters.

III. CONJUGATE METHOD

Exercising Bayesian inference represented in (12) is quite straightforward, but exercising (13) is not. In practice, the integral in the denominator of (13) is intractable. The conjugate method is a tool to circumvent that intractable integral which is replaced by analytic algebraic equations [16].

Unfortunately, the conjugate method is not applicable to all cases. It is confined to some special cases. For example, if research topic data comply with binomial distribution, and if beta distribution is selected to suggest binomial distribution parameter, then the beta-binomial conjugate method can be applicable. Another example is that if the research topic data comply with normal distribution, and if a normal distribution is selected to suggest the former normal distribution's mean μ, then normal–normal conjugate method is applicable. The normal–normal conjugate method requires that the former normal distribution' standard deviation σ shall be fixed.

This paper assumes that the renewable energy generation data comply with the normal distribution. Exercising Bayesian inference to estimate parameter μ, we have

TABLE I.
STEPS FOR BAYESIAN INFERENCE

Step	Content
1	Assume a probability distribution that suits the research topic.
2	Suggest hypotheses, i.e., parameters, of the assumed probability distribution in step 1.
3	Select another probability distribution for each hypothesis (i.e., parameter) in step 2. This selected probability distribution is called the prior probability distribution whose parameters are called hyperparameters.
4	Collect data regarding the research topic.
5	Calculate likelihood of the collected data under each hypothesis.
6	Update hyperparameters of the prior probability distribution by (8) or (9). The updated probability distribution becomes the posterior probability distribution.
7	Infer the probability distribution from the assumed one in step 1 with the aid of the posterior probability distribution.

$$P(\mu|\text{data}) = \frac{P(\text{data}|\mu)\cdot P(\mu)}{\int P(\text{data}|\mu)\cdot P(\mu)d\mu} \tag{14}$$

To circumvent the intractable integral in the denominator of (14), the normal–normal conjugate method is summoned up. First, a statistical term called precision, denoted by τ, is defined by

$$\tau = \frac{1}{\sigma^2} \tag{15}$$

Or namely,

$$\sigma = \sqrt{\frac{1}{\tau}} \tag{16}$$

Next, the normal–normal conjugate method updates the hyperparameters of the prior probability distribution to

$$\mu_{posterior} = \frac{\tau_{prior}\mu_{prior} + \tau\sum_{i=1}^{n} x_i}{\tau_{prior} + n\times\tau}, \tag{17}$$

$$\tau_{posterior} = \tau_{prior} + n\times\tau \tag{18}$$

Note that *n* and X_i represent the total number of collected data and the i^th^ data sample, respectively. τ is the precision of the assumed normal distribution. Notice that in (17) and (18), the subscript "prior" denotes the hyperparameters of prior probability distribution, and the subscript "posterior" denotes those of posterior probability distribution. Table II summarize the steps of Bayesian inference with normal–normal conjugate method.

TABLE II.
STEPS FOR BAYESIAN INFERENCE WITH NORMAL–NORMAL CONJUGATE METHOD

Step	Content
1	Assume a probability distribution that suits the research topic.
2	Suggest hypotheses, i.e. parameters, of the assumed probability distribution in step 1.
3	Select another probability distribution for each hypothesis (i.e., parameter) in step 2. This selected probability distribution is called the prior probability distribution whose parameters are called hyperparameters.
4	Collect data regarding the research topic.
5	Update hyperparameters of the prior probability distribution by (13) and (14) to obtain hyperparameters of the posterior probability distribution.
6	Infer the probability distribution from the assumed one in step 1 with the aid of the posterior probability distribution.

IV. CASE STUDY

A. Data Description

This paper considered the waste-to-energy generation data in Taiwan from January 2018 to October 2023. These data can be found from the Taiwan government open data platform [17]. These data were selected because the waste-to-energy equipment total capacity in Gigawatt over this time span was fixed. Table III shows parts of these data. The first column is the year and month. The second column is the total capacity of waste-to-energy power generation equipment [18]. The unit is Gigawatt. The third column is the electric power generation of that month in Taiwan [19]. Its unit is Gigawatt hours.

B. Bayesian Inference Results

We followed the steps listed in Table II. Step 1 assumed a normal distribution could represent waste-to-energy power generation monthly in Taiwan. Step 2 suggested that the assumed probability distribution mean μ was 367.5374 GWH and standard deviation σ was 50 GWH. The suggested mean μ came from the total operating hours at RenWu waste-to-energy plant in 1 year were 7830 hours, and the ratio of equipment utilization was 89.14% [20]. Accordingly, the mean μ was suggested by

TABLE III.
WASTE-TO-ENERGY GENERATION MONTHLY IN TAIWAN

Date	Capacity (GW)	Generation (GWH)
01/2018	0.6319	335.8172
02/2018	0.6319	300.4179
03/2018	0.6319	296.3097
...	...	...
10/2023	0.6319	265.9801

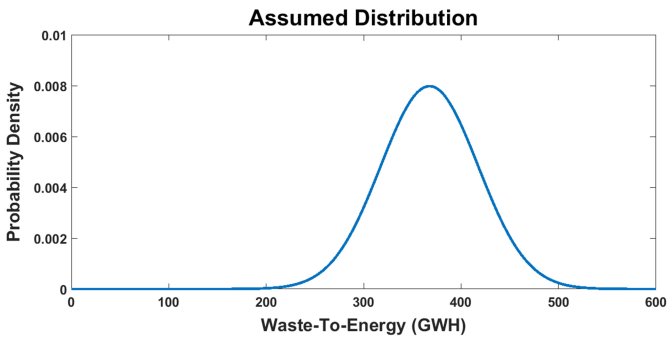

Fig. 3. Assumed probability distribution for waste-to-energy generation.

$$0.6319 \times 7830 \times 0.8914 \div 12 = 367.5374 \text{ GWH per month} \quad (19)$$

Fig. 3 shows the assumed probability distribution for monthly waste-to-energy power generation monthly in Taiwan. Notice that the standard deviation σ must be fixed as we used normal–normal conjugate method.

Step 3 selected another normal distribution, i.e., prior distribution, to represent the parameter mean μ. The hyperparameters μ_{piror} and σ_{piror} were set to 367.5374 GWH and 70 GWH, respectively. The μ_{piror} was set 367.5374 GWH because of the same suggestion in the last paragraph, while the σ_{piror} was set 70 GWH to allow a larger range. Fig. 4 plots the prior distribution for μ.

Step 4 collected 60 data samples from 01/2018 to 12/2022 in Table III. Step 5 updated the posterior distribution hyperparameters $\mu_{postetior}$ and $\sigma_{posterior}$ from (17) and (18). It yielded 300.2529 for $\mu_{posterior}$, and 6.4277 for $\sigma_{posterior}$. The posterior distribution is plotted in red dotted line in Fig. 5. To compare, Fig. 5 placed the prior and posterior distributions in one figure. It can be seen that the posterior distribution has been shifted toward left, and it has also been centralized. Fig. 5 suggests that we are more confident in the parameter μ, the aid of Bayesian inference normal–normal conjugate method and also of the collected data.

Once we had updated hyperparameters $\mu_{posterior}$ and $\sigma_{posterior}$ for the posterior distribution, we applied the z-test to find the confidence interval of the parameter μ [21]. Calculate the z value.

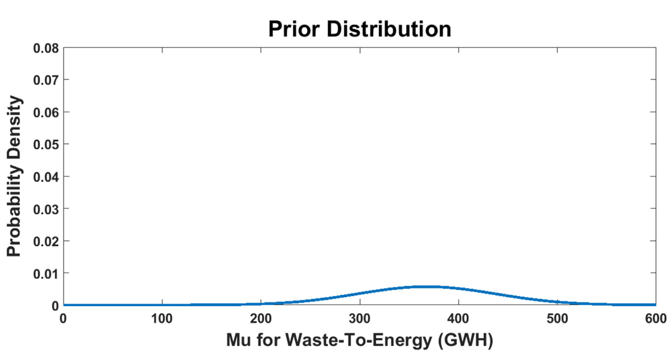

Fig. 4. Prior distribution for μ.

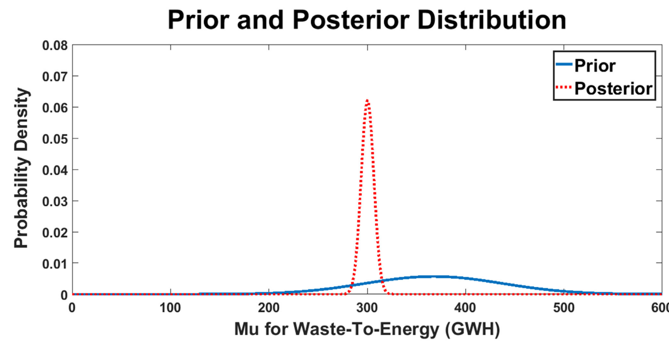

Fig. 5. Prior and posterior distribution for μ.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \quad (20)$$

Considering two-sided confidence level of 99%, we found from standard normal distribution table that the critical z value is 2.58. Therefore, (20) led to

$$-2.58 \le \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \le 2.58 \quad (21)$$

Substituting $\mu_{posterior}$ 300.2529 and $\sigma_{posterior}$ 6.4277 into (21), we obtained

$$300.2529 - \frac{2.58 \times 6.4277}{\sqrt{60}} \le \mu \le 300.2529 + \frac{2.58 \times 6.4277}{\sqrt{60}} \quad (22)$$

Rearranging (22), we had

$$298.1120 \le \mu \le 302.3938 \quad (23)$$

Above inequality states that the probability of parameter μ falling in the interval of 298.1120 and 302.3938 is 0.99, with the aid of the posterior distribution and z-test.

Finally, step 6 infers the probability distribution. Adopting the minimum μ and maximum μ updated in step 5, and together with the σ suggested in step 2, we inferred the distribution for waste-to-energy generation monthly in Taiwan as plotted in red dotted curve in Fig. 6.

To find the confidence interval of 99% for waste-to-energy generation, we performed z-test again. Using minimum μ updated in step 5 and σ suggested in step 2, we have

$$298.1120 - \frac{2.58 \times 50}{\sqrt{60}} \le \text{generation} \le 298.1120 + \frac{2.58 \times 50}{\sqrt{60}} \quad (24)$$

Therefore, (24) yielded

$$281.4582 \le \text{generation} \le 314.7658 \quad (25)$$

Using maximum μ updated in step 5 and σ suggested in step 2, we have

$$302.3938 - \frac{2.58 \times 50}{\sqrt{60}} \le \text{generation} \le 302.3938 + \frac{2.58 \times 50}{\sqrt{60}} \quad (26)$$

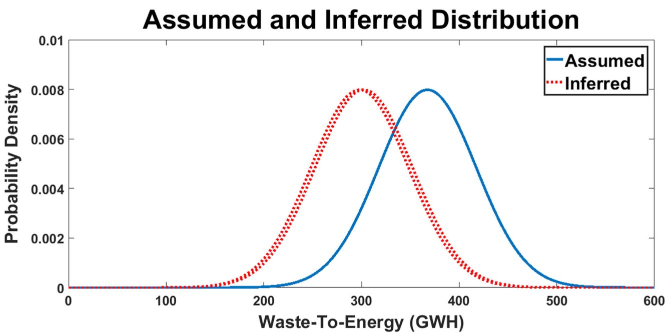

Fig. 6. Assumed and inferred distributions for waste-to-energy generation.

Therefore, (21) yielded

$$285.7400 \leq \text{generation} \leq 319.0476 \tag{27}$$

According to (25) and (27), we concluded that the inferred distribution of waste-to-energy generation with a 99% confidence level was

$$281.4582 \leq \text{generation} \leq 319.0476 \tag{28}$$

C. Tests

Recalled the assumed probability distribution was μ 367.5374 and σ 50. Again, by setting a 99% confidence interval in the z-test, the assumed probability distribution of waste-to-energy generation was

$$367.5374 - \frac{2.58 \times 50}{\sqrt{60}} \leq \text{generation} \leq 367.5374 + \frac{2.58 \times 50}{\sqrt{60}} \tag{29}$$

Therefore, the assumed distribution forecasted that, with 99% confidence, the generation was within the interval

$$350.8836 \leq \text{generation} \leq 384.1912 \tag{30}$$

While the inferred distribution forecasted that, with 99% confidence, the generation was within the interval

$$302.3938 - \frac{2.58 \times 50}{\sqrt{60}} \leq \text{generation} \leq 302.3938 + \frac{2.58 \times 50}{\sqrt{60}} \tag{31}$$

Table IV shows the unforeseen 10 data samples testified by assumed and inferred probability distributions. It can be seen that the assumed probability distribution failed for all unforeseen data samples, while the inferred probability distribution covers eight of the ten unforeseen data samples. The accuracy is 80%.

V. DISCUSSION

In this paper, the forecast was presented by a probability distribution. Using this probability distribution, we could forecast the generation with a certain confidence level within an interval. The forecast claimed that, with a 99% confidence level, the interval between 281.4582 GWH and 319.0476 GWH could include the monthly generation of waste-to-energy in Taiwan.

TABLE IV.
WASTE-TO-ENERGY GENERATION TEST BY ASSUMED AND INFERRED DISTRIBUTION

Date	Generation (GWH)	Assumed	Inferred
01/2023	298.4644	X	√
02/2023	284.8347	X	√
03/20233	288.2910	X	√
04/2023	262.6507	X	√
05/2023	306.7749	X	√
06/2023	297.2915	X	√
07/2023	314.0844	X	√
08/2023	304.9642	X	√
09/2023	283.1953	X	√
10/2023	265.9801	X	√

X, failed; √, successful.

If the confidence interval's lower limit is set at 0 and higher limit at 10 000 GWH, all forecast values shall fall within this interval, and one may claim a 100% success rate accordingly. However, such a 100% successful forecast might be mistakenly interpreted. A higher confidence level leads to a wider interval, but it is less precise [22]. It is not informative, either. On the contrary, a lower confidence level leads to a smaller interval, but is more precise. It is more informative.

Although researchers may consider different confidence levels at will, in literature 95% and 99% are the most prevalent. A higher confidence level is usually reserved for those situations where a false confidence interval might cause critical consequences. Accordingly, this paper considered a 99% confidence level.

VI. CONCLUSION

This paper applies Bayesian inference with normal–normal conjugate method to forecast intermittent renewable energy generation. An assumed probability distribution is initialized for renewable energy generation forecast. Bayesian inference with normal–normal conjugate method infers the probability distribution. A case study of waste-to-energy renewable power generation in Taiwan is investigated. Numerical tests demonstrate that the inferred probability distribution has 80% accuracy. The inferred probability distribution outperforms the assumed counterpart. The forecast accuracy might be improved further since the normal–normal conjugate method has to fix σ in advance. Therefore, if σ can be released and inferred as well as μ, the renewable energy generation forecast might be more accurate. In addition, the proposal is expected to be applicable to other intermittent renewable energy sources forecast. These will be the author's future works.

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